NOTICE

THIS DOCUMENT HAS BEEN REPRODUCED FROM MICROFICHE. ALTHOUGH IT IS RECOGNIZED THAT CERTAIN PORTIONS ARE ILLEGIBLE, IT IS BEING RELEASED IN THE INTEREST OF MAKING AVAILABLE AS MUCH INFORMATION AS POSSIBLE

NVSV

Technical Memorandum 81998

Inverting x, y Grid Coordinates to Obtain Latitude and Longitude in the Van Der Grinten Projection

David Parry Rubincam

(NASA-TM-81998) INVERTING X, Y GRID COORDINATES TO OBTAIN LATITUDE AND LONGITUDE IN THE VANDERGRINTE" PROJECTION (NASA) 13 P HC A02/MF A01 CSCL 08B

N81-12528

Unclas G3/43 39831

AUGUST 1980

National Aeronautics and Space Administration

Goddard Space Flight Center Greenbelt, Maryland 20771



INVERTING x, y GRID COORDINATES TO OBTAIN LATITUDE AND LONGITUDE IN THE VAN DER GRINTEN PROJECTION

David Parry Rubincam

Geodynamics Branch
Code 921
Goddard Space Flight Center
Greenbelt, Maryland 20771

August 1980

INVERTING x, y GRID COORDINATES TO OBTAIN LATITUDE AND LONGITUDE IN THE VAN DER GRINTEN PROJECTION

David Parry Rubincam

ABSTRACT

The latitude and longitude of a point on the earth's surface are found from its x, y grid coordinates in the van der Grinten projection. The latitude is a solution of a cubic equation and the longitude a solution of a quadratic equation. Also, the x, y grid coordinates of a point on the earth's surface can be found if its latitude and longitude are known by solving two simultaneous quadratic equations.

INVERTING x, y GRID COORDINATES TO OBTAIN LATITUDE AND LONGITUDE IN THE VAN DER GRINTEN PROJECTION

Interest in the van der Grinten projection (van der Grinten, 1905) has revived recently because of its use in the geophysical atlas of Lowman and Frey (1979). The maps contained therein are based in turn on the map "The Physical World" (1977 revision), which is produced by the National Geographic Society and uses the van der Grinten projection.

O'Keefe and Greenberg (1977) have given an algorithm for doing the forward problem in the van der Grinten projection: showing how to go from latitude and longitude to x, y grid coordinates.

Rubincam (1980b) has made corrections to some of the equations given in O'Keefe and Greenberg (1977). Snyder (1979) also made a correction.

In this paper we will show how to do the inverse problem; going from x, y grid coordinates to latitude and longitude. This is required for some applications. For instance, Rubincam (1980a) computes a density distribution based on the gravity field at points on a rectangular array in grid coordinate space (see Fig. 1) for plotting on the tectonic activity map of P. Lowman (Lowman and Frey, 1979, p. 40) in order to observe correlations between density and tectonics. Since the density distribution is expressed in terms of spherical harmonics, the latitude and longitude are necessary for computing the density at each array point; hence the need for the inversion.

Further, we will give an alternative algebraic algorithm for doing the forward problem, which may be simpler than the trigonometric approach of O'Keefe and Greenberg (1977). Snyder (1979) has also given an alternative algorithm.

In the following we will use the notation of O'Keefe and Greenberg (1977).

We first wish to find latitude ϕ and longitude λ of a point when the grid coordinates (x, y), reference longitude λ_0 , and scale factor ρ are known. The origin of the grid coordinate system is at $\phi = 0$ and $\lambda = \lambda_0$.

Let us begin with the latitude ϕ . We will assume that y > 0, so that $\phi > 0$ and b > 0, where $\phi = b\pi$ and ϕ is measured in radians. It proves convenient to find b from

$$x^2 + (y - y_1)^2 = s^2,$$
 (1)

which is the equation for a circle (see Fig. 2). Quantities y_j and s can be expressed in terms of b and ρ :

$$y_{J} = \frac{2\rho (1-b)\sqrt{1+2b}}{b(\sqrt{1+2b}-\sqrt{1-2b})}$$
 (2)

$$s = \frac{2\rho (1+b)\sqrt{1-2b}}{b(\sqrt{1+2b}-\sqrt{1-2b})}$$
(3)

These two equations are derived in the Appendix. They agree with those originally given by van der Grinten (1905, p. 360) for 2b = c and $2\rho = 1$.

Substituting (2) and (3) in (1) and multiplying by

$$b(\sqrt{1+2b}-\sqrt{1-2b})^2$$

gives

$$[4\rho(1-b)y-2b(x^2+y^2)]\sqrt{1-4b^2}=4\rho(1-b)(1+2b)y-2b(x^2+y^2)-16\rho^2b^2.$$

Squaring this expression and dividing by 16b gives the cubic equation

$$c_3b^3 + c_2b^2 + c_1b + c_0 = 0 (4)$$

after considerable algebra, where

$$c_3 = 16\rho^4 + 16\rho^3y + 8\rho^2y^2 + 4\rho y(x^2 + y^2) + (x^2 + y^2)^2$$

$$c_2 = -8\rho^3y + 4\rho^2(x^2 + y^2) - 12\rho^2y^2 - 2\rho y(x^2 + y^2)$$

$$c_1 = -8\rho^3y - 2\rho y(x^2 + y^2)$$

$$c_0 = 4\rho^2y^2.$$

and

The trigonometric solution is the most convenient way to solve (4). It is (Selby, 1973, pp. 103-104):

$$b = m \cos \left(\theta + \frac{4\pi}{3}\right) - \frac{c_2}{3c_3} \tag{5}$$

where

 $\theta = \frac{1}{3} \operatorname{Arc} \cos \left(\frac{3d}{am} \right)$

$$m = 2\sqrt{\frac{a}{3}}$$

with

 $d = \frac{1}{27} \left(2 \frac{c_2^3}{c_3^3} - 9 \frac{c_2 c_1}{c_3^2} + 27 \frac{c_0}{c_3} \right)$

and

$$a = \frac{1}{3} \left(3 \frac{c_1}{c_3} - \frac{c_2^2}{c_3^2} \right).$$

Latitude ϕ is then found by multiplying (5) by π . This is the desired result for y > 0. The latitude ϕ when y < 0 is easily found from the above result, since the van der Grinten projection has reflective symmetry about the equator. In this case we merely set y = -y, solve for b from (5), and then set $\phi = -b\pi$. The case for y = 0 is trivial; here $\phi = b = 0$.

The longitude λ can be found from the equation (see Fig. 2)

$$(x - x_M)^2 + y^2 = r^2, (6)$$

where (O'Keefe and Greenberg, 1977; Rubincam, 1980b):

$$x_{M} = \rho \left(\ell - \frac{1}{\ell} \right), \tag{7}$$

$$r = \left| \rho \left(\Omega + \frac{1}{\Omega} \right) \right| , \qquad (8)$$

and

$$\ell = \frac{\lambda - \lambda_0}{\pi} \ . \tag{9}$$

Substituting (7) and (8) in (6) gives an equation quadratic in ℓ , with solution

$$\varrho = \frac{x^2 + y^2 - 4\rho^2 + \sqrt{16\rho^4 + 8\rho^2(x^2 - y^2) + (x^2 + y^2)^2}}{4\rho x}$$
(10)

when $x \neq 0$. Longitude λ is found from (10) via (9). The trivial case of x = 0 gives $\lambda = \lambda_0$.

This completes our derivation of the basic equations for the inversion process. They have been verified through numerical examples.

The forward problem can be done according to the following algorithm. First, find b and ℓ from ϕ , λ , and λ_0 . Next, compute y_J by substituting |b| for b in (2) and putting a minus (-) sign in front of the resulting expression if b < 0. Then compute s by substituting |b| for b in (3). After that, compute x_M and r from (7) and (8). Finally, find grid coordinates x and y by solving simultaneously the quadratic equations (1) and (6). This gives

$$x = \frac{x_{M}(x_{M}^{2} + y_{J}^{2} + s^{2} - r^{2}) + \frac{\ell}{|\ell|} |y_{J}| \sqrt{4r^{2}s^{2} - [(x_{M}^{2} + y_{J}^{2}) - (r^{2} + s^{2})]^{2}}}{2(x_{M}^{2} + y_{J}^{2})}$$
(10)

$$y = \frac{y_{J}(x_{M}^{2} + y_{J}^{2} + r^{2} - s^{2}) - \frac{b}{|b|} |x_{M}| \sqrt{4r^{2}s^{2} - [(x_{M}^{2} + y_{J}^{2}) - (r^{2} + s^{2})]^{2}}}{2(x_{M}^{2} + y_{J}^{2})}$$
(11)

These equations have been verified numerically.

Our approach to the forward problem may be quicker than the trigonometric approach of O'Keefe and Greenberg (1977) when programmed on a computer. It certainly involves less confusion about signs. Further, it differs from Snyder's (1979) approach; his involves some trigonometry, while ours is wholly algebraic.

Summarizing our results on the van der Grinten projection, we have: verified van der Grinten's original equations for y_j and s, which are given by our (2) and (3); solved the inverse problem of going from x, y grid coordinates to latitude and longitude; and given an algebraic algorithm for doing the forward problem of going from latitude and longitude to x, y grid coordinates.

APPENDIX

Derivation of Equations for s and y,

Here we derive (2) and (3) to find s and y_j in terms of b and ρ . We make constant reference to the geometrical relationships of Fig. 2. Our Fig. 2 is based on O'Keefe and Greenberg's (1977) Fig. 2.

Now A'O = 2ρ and BO = 4ρ b (O'Keefe and Greenberg, 1977). Also, FO = BO - BF = 4ρ b - BF. Since BEN is a right isoseles triangle, we have BE = BN = $2\rho - 4\rho$ b. These equations and the ratios of the lengths of sides of similar triangles BEF and A'FO give

$$BF = \frac{2\rho b (1-2b)}{1-b}$$
.

From right triangle BCO and the Pythagorean theorem we have

$$BC = 2\rho \sqrt{1 - 4b^2}$$

Also, DO = $4\rho b$ – BD. So by similar triangles BCD and A'DO we get

$$\frac{BD}{BC} = \frac{DO}{2a} = \frac{4\rho t - BD}{2a}$$

or

$$BD = \frac{4\rho b \sqrt{1 - 4b^2}}{1 + \sqrt{1 - 4b^2}}.$$

Therefore

DF = BD - BF =
$$\frac{2\rho b(-1 + 2b + \sqrt{1 - 4b^2})}{(1 - b)(1 + \sqrt{1 - 4b^2})}$$
.

We have also FO = $2\rho b/(1 - b)$. From right triangle FGO we obtain

$$FG = \frac{2\rho\sqrt{1-2b}}{1-b},$$

and from right triangle DFG we get

$$(DG)^2 = \frac{8\rho^2(1+b)(1-2b)}{(1-b)(1+\sqrt{1-4b^2})},$$

both by Pythagoras.

Now by similar triangles DHJ and DFG we have

$$\frac{DJ}{\frac{1}{2}DG} = \frac{DG}{DF}$$

where DH = DG/2. But DJ = s; so

$$s = \frac{(DG)^2}{2DF} = \frac{2\rho(1+b)\sqrt{1-2b}}{b(\sqrt{1+2b}-\sqrt{1-2b})},$$

which is the desired equation for s.

Finally, $y_J = OJ = DJ + DO = s + 4\rho b - BD$, so that

$$y_{J} = \frac{2\rho(1-b)\sqrt{1+2b}}{b(\sqrt{1+2b}-\sqrt{1-2b})},$$

which is the desired equation for y_I.

References

- Lowman, P. D., and Frey, H. V. (1979), editors, "A geophysical atlas for interpretation of satellitederived data," NASA TM 79722, Goddard Space Flight Center.
- O'Keefe, J. A., and Greenberg, A. (1977), "A note on the van der Grinten projection of the whole earth onto a circular disk," Amer. Cartographer, 4, 127-132.
- Rubincam, D. P. (1980a), "Information theory density distribution," in "Earth Survey Applications

 Division Research Report-1979," edited by Lloyd Carpenter, NASA TM 80642, Goddard Flight

 Center, pp. 5-4 to 5-12.
- Rubincam, D. P. (1980b), "Errata for 'A note on the van der Grinten projection of the whole earth onto a circular disk' by John A. O'Keefe and Allen Greenberg," Amer. Cartographer, in press.
- Selby, S. M. (1974), editor, <u>Standard Mathematical Tables</u>, Twenty-second edition, CRC press, Cleveland, Ohio.
- Snyder, J. P. (1979), "Projection notes," Amer. Cartographer, 6, 81.
- van der Grinten, A. (1905), "New circular projection of the whole earth's surface," Am. Jour. Sci., 19, 357-366.

	######################################	
	AAAAAARI JAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA	
#12 · · · * # #.	ALLELELELELELELELELELELELELELELELELELEL	
	1818711879111831118444450000000000000000000000000000000	_
	######################################	_
		 -
-	IPPERILIPAN TOLLY FERNAN AND AND AND AND AND AND AND AND AND	
	12767111110010 7000 0000000000000000000000	
	HITTITIA AAAAAAAAAAAAA IIIIIAA ABBUUA A AIIIAA. W.U. SAAA AAA TAA TABIIIIII AAAAAAAAAAAAAAAAAA	
	#ATABABA BERT PARALIFER BANGALIFER TARAKA APAR 1 11-01A7 111A NO 150-1111 1111 111 111 111 111 111 111 111	
	1111114AAA-LEFAAA1111MAAATIIIIIIAAAHUAAAPPAACELEAAIIIII PARICHEMPAAAAAULPIAAAAIIIIIIIIIIIIIIIIIIIIIIIIII	
	44 * 44 11 11 11 11 11 11	
	######################################	
	AA 11111111111111111111111111A** ABMAKABABAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA	•
	111411112711111111111111111111111111111	
	*#####################################	
	AN CHANTIER TO A CONTROL OF THE PROPERTY OF TH	
مرجست مستني	TUPPITISTIPETRIPPTZ.UTZTANIMULTPTIPRAPAAATULANAAAAAAAAAAAAAAAATIAAAAAAAAAAAAAAAAAAA	
	142222/16461464/161111111111111111111111	
	AD-BARAKARA 13:13:20-21:40 1 22:20-21:40 10:00-21:40 1	
	MAANIFET MAANIFAAAAAAT 2714AAAAT AAAAAT AAAAT AAAT AAAAT 1800 FUAR 568 12 F 6 AATTI TAAAAAT 1855 AATTA AAAAT AAAT AAAT AAAT AAAAT AA	
	BARTETTE TEAT A TELETITETA SHITA E TARAAAN GU GO COLATE AL GO LA GRAFET E TEAT LA TELETA AL AL TELATO CALLET E CALIFIEL LANGE E TELETITET ATTARA MILLANDE PARTICO PERTICO PER LO CALIFE E LA CALIFET E AL ARTES PARA EL PARTE	
	######################################	
	AAAFTAATTITTAEZAAAAAATTAAA HAAATTITTAAAAAAAAA - LAAAAAAATTITTTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTTAATTITTAATTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTAATTITTA	
	######################################	
	Allananani langananananan an ilisa ililili ili pina anananananan menahangan 1777/1 anan ilili janah ilaa 12 37 11 4 4 11 11 12 12 12 12 12 12 12 12 12 12 12	
	1111 1846 1751 11151 14 141 1861 141 1861 181 181 181 181 181 181 1864 181 1864 181 1866 1866	
	######################################	
	AAAAAAAAAAAA1114AAAAAA4111221337AA153135313131313131313140AAAAAAAAAAAAAAAAA112633313135127514AAAAAAAA3332, 1 AAAAA1333331313494441333331331331331313131313	
	AAAAAAAAAAAAAAAA 11 22 11 12 12 13 14 14 14 14 14 14 14 14 14 14 14 14 14	
	######################################	
	######################################	
	### ##################################	
	#MDFC644AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA	
	military was a second of the s	-
	TITITIAN THE CONTROL OF THE CONTROL	
	- 1911111114AAAAAAAAAA 1111777771114AAAAAAAAAA	
	######################################	
	######################################	
	AND	
100		
	MANAGA A 183 1 3 2 3 3 1 1 1 2 3 1 1 2 2 3 3 3 3 3	~-
	Bibhanan 11111 20027	
	MINDAAAAA 1111172 WITTE BUTTE WITTE WALLE WAAAAA GERMAAAAA GERMAAAAAA GERMAAAAAA GERMAAAAAAA GERMAAAAAAA GERMAAAAAAA GERMAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA	-

Figure 1. Example of a computer plot of the information theory density distribution derived from the gravity field. Only part of the eastern hemisphere is shown. Letters represent low density and numbers high density.

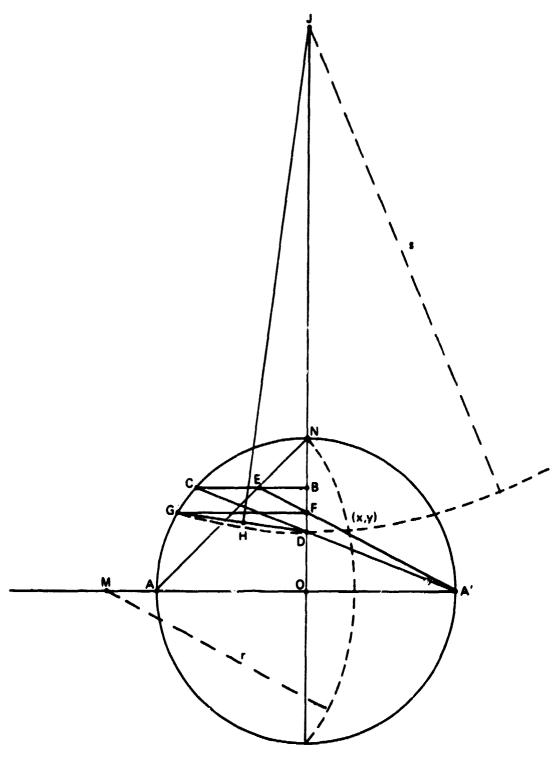


Figure 2. Geometrical relationships in the van der Grinten projection based on O'Keefe and Greenberg (1977). Note that the coordinate point (x, y) is found from the intersection of two circles.

BIBLIOGRAPHIC DATA SHEET

1. Report No. 81998	2. Government Acc	ession No. 3.	Recipiont's Catalo	No.			
4. Title and Subtitle	Ola 1 - 1 - 1 - 1	5. Report Date August 1980					
Inverting x, y Grid Coordinates Longitude in the Van Der Grin			6. Performing Organization Code 921				
7. Author(s) David Parry Rubincam		8.	Performing Organi	zation Report No.			
Performing Organization Name an Geodynamics Branch	d Address	10	. Work Unit No.				
Code 921 Goddard Space Flight Center		11	. Contract or Gran	t No.			
Greenbelt, Maryland 12. Sponsoring Agency Name and A	ddress	13	. Type of Report a	nd Period Covered			
NASA/Goddard Space Flight (Greenbelt, Maryland		Technical Memorandum					
Greenbert, maryland	14	. Sponsoring Agend	cy Code				
15. Supplementary Notes Submitted to The American Cartographer							
16. Abstract The latitude and longitude of a point on the carth's surface are found from its x, y grid coordinates in the van der Grinten projection. The latitude is a solution of a cubic equation and the longitude a solution of a quadratic equation. Also, the x, y grid coordinates of a point on the earth's surface can be found if its latitude and longitude are known by solving two simultaneous quadratic equations.							
17. Key Words (Selected by Author		18. Distribution Statement					
Van der Grinten projection							
19. Security Classif. (of this report)	20. Security Class	if. (of this page)	21. No. of Pages	22. Price*			
Unclassified	Unclassified	-	11				
For sale by the National Technical Inform	etion Service. Scringlie	d. Virginia 22151.	البيدة البيونيين عاديها	GSFC 25-44 (16/77)			